Collective Adoption of Max-Min Strategy in an Information Cascade Voting Experiment

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Abstract

When one have to choose an option, he shows a strong tendency to choose the majority if he does not know the correct one. If each option has a multiplier m and the return for choosing a correct choice is set to be m, which option does one choose? Game theory predicts that the max-min strategy where one divides one's choice inversely proportional to m is optimal. We study the prediction by a voting experiment in which 50 to 60 subjects answer a two-choice quiz sequentially with and without information about prior subjects' choices. The information is given to the subjects in two patterns, C and M. In case C, the subjects know how many previous subjects have chosen each choice and the payoff for the correct choice is constant. In case M, each choice has a multiplier m that is inversely proportional to the number of prior subjects who have chosen it. The payoff for the correct choice is proportional to the multiplier. In case C, the probability of selecting a choice by the subject who did not know the correct choice rapidly increases as the proportion of subjects who have chosen it increases. In case M, the probability is inversely proportional to m for $4/3 \leq m \leq 4$. The subjects collectively adopt the Max-Min strategy in the range. The threshold value of the information cascade phase transition increases considerably as compared to in case C.

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I. INTRODUCTION

Even if each person has limited information, aggregated information becomes very accurate [1]. This is the wisdom of crowd effect, and it is supported by many examples from political elections, sports predictions, quiz shows, and prediction markets [2–4]. In contrast, in order to give accurate results, three conditions need to be satisfied: diversity, independence, and decentralization. If these conditions are not satisfied, aggregated information becomes unreliable or worse [2, 5]. However, in an ever-more connected world, it becomes more and more difficult to retain the independence. Furthermore, if the actions or choices of others are visible, neglecting them is not realistic in light of the merit of social learning [6, 7]. In this case, information cascade may emerge and information aggregation ceases [8–14].

More concretely, we consider a situation where people answer a two-choice question with choices A and B sequentially. Before this question is asked, many other people have already answered and their choices are made known as C_A people choosing A and C_B people choosing B, which is called social information. If the person answering knows the correct choice, he should choose it. His choice is not affected by social information. We then call him an independent voter. However, if he does not know the correct choice, he will be affected by social information [15]. He tends to go with the majority, and this is rational behavior. We then call him a herder, because he copies the majority. By rational herding, the wisdom of crowds is on the edge. If a herder is isolated from others, his choice becomes A and B and should be canceled. As a result, the choice by an independent voter remains. The majority choice always converges to the correct one in the limit of a large number of people. This is known as Condorcet's jury theorem [1]. However, if others' choices are given as social information, the cancellation mechanism does not work. The herder copies the majority and ignores the correct information given by the independent voter. If the proportion of herders p exceeds some threshold value p_c, there occurs a phase transition from the one-peak phase where the majority choice always converges to the correct one to the two-peak phase where the majority choice converges to the wrong one with a finite and positive probability [16]. We call this phase transition information cascade transition [17, 18]. This is the risk of imitation in the wisdom of crowd. How can we avoid this risk? There exists a hint in race-track betting markets and prediction markets [19]. In order to aggregate information scattered among people, the market mechanism or an invisible hand can be very effective [20].

We consider a situation in which each choice $\alpha \in \{A, B\}$ has a multiplier M_α that is inversely proportional to the number of subjects C_α who chose it. The payoff for the correct choice is proportional to the multiplier. If the multiplier of a choice is large, the number of people who chose it is small. If the return is constant, it is not rational for a herder to choose the choice. However, now, the return on the correct choice is proportional to the multiplier, and hence we cannot say that it is not rational. Copying the majority gives him a small return, even if it is a correct choice. The multiplier plays the role of "tax" for herding (free rider) and copying the minority can be an attractive choice. By a Max-Min argument based on game theory [21], we can show that an optimized behavior is the one where a herder chooses α with a probability proportional to C_α. We call the herder who adopts the optimized behavior an analog herder [22]. If the herder behaves an analog herder, the information cascade phase transition does not occur. Instead of the phase transition, the convergence speed phase transition occurs as the proportion of analog herders exceeds half

[23]. The derivative of the convergence rate becomes discontinuous at this point. However, the majority of people always choose the correct choice in the limit of a large number of people and the system is in the one-peak phase for any value of the proportion. An invisible hand induced by multipliers automatically removes the risk of imitation in the wisdom of crowd.

In this paper, we have adopted an experimental approach to study whether herders collectively adopt the optimized strategy and behave as analog herders if the choices have multipliers. We have also studied the information cascade transition of the system and the performance of the wisdom of crowd. The organization of the paper is as follows. We explain the experiment and the optimized strategy in section II. The subjects answer a two-choice quiz in three cases $r \in \{O, C, M\}$. In case O, the subjects answer without social information. In cases C and M, they receive social information based on previous subjects' choices. Social information is given as summary statistics $\{C_A, C_B\}$ in case C and as multipliers $\{M_A, M_B\}$ in case M. Sections III and IV are devoted to the analysis of the experimental data. In section III, we study the macroscopic aspects of the system. As the proportion of herders approaches 100%, the convergence of the sequence of choices becomes extremely slow and information aggregation almost ceases in both cases $r \in \{C, M\}$. In section IV, we derive a microscopic rule regarding how herders copy others in each case $r \in \{C, M\}$. In section V, we introduce a stochastic model that simulates the system. We study the information cascade phase transition and the performance of herders in the system. Section VI is devoted to the summary and discussions. In the appendices, we give some supplementary information about the experiment and the estimation procedure of the parameters in section IV.

II. EXPERIMENTAL SETUP AND OPTIMIZED SRATEGY IN CASE M

A. Experimental design

The experiment reported here was conducted at the Group Experiment Laboratory of the Center for Experimental Research in Social Sciences at Hokkaido University. We have conducted two experiments and call them EXP-I and EXP-II. In EXP-I (II), we recruited 120 (104) students from the university. We divided them into two groups, Group A and Group B, and prepared two sequences of subjects of average length 60 (50). The subjects answered a two-choice quiz of 120 questions sequentially. We label the questions by $i \in \{1, 2, \cdots, 120\}$.

In EXP-I, the subject answers in three cases $r \in \{O, C, M\}$ in this order. We denote the answer to question i in case r after $t-1$ subjects' answers by $X(i, t|r)$, which takes the value 1 (0) if the choice is true (false). $\{C_0(i, t|r), C_1(i, t|r)\}$ are the numbers of subjects who choose true and false for question i among the prior t subjects as

$$C_1(i, t|r) = \sum_{t'=1}^{t} X(i, t'|r),$$
$$C_0(i, t|r) = t - C_1(i, t|r). \tag{1}$$

In case O, the subject answered without any social information. Then, he answered in case C. Before him, $t-1$ subjects answered question i, and he received summary statistics $\{C_A(i, t{-}1|C), C_B(i, t{-}1|C)\}$ from them. For the correct choice in cases O and C, the subject gets two points. Finally, in case M, the subject receives multipliers $\{M_A(i, t{-}1), M_B(i, t{-}1)\}$

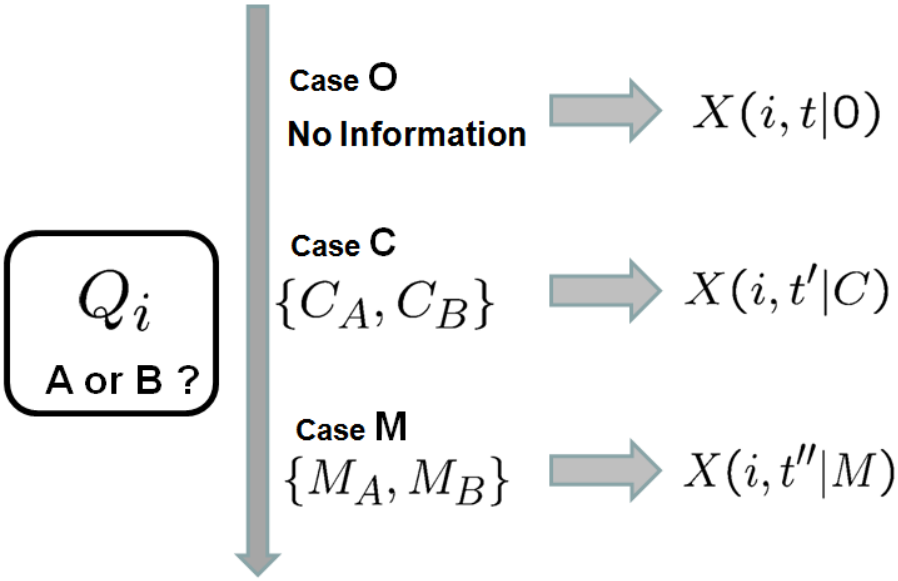

FIG. 1. Pictorial explanation of the experimental procedure. There are 120 questions in the quiz, labeled by $i \in \{1, 2, \cdots, 120\}$. A subject answers question i in three cases $r \in \{O, C, M\}$ in this order. If the answer was given after $t-1$ subjects, it is denoted as $X(i,t|r)$.

from all previous $t-1$ subjects. For the correct choice, the subject gets the points which is given by the multiplier. The multiplier M_α for $\alpha \in \{A, B\}$ was calculated based on the summary statistics in case M as

$$M_\alpha(i, t-1) = \frac{C_A(i, t-1|M) + C_B(i, t-1|M) + 1}{C_\alpha(i, t-1|M) + 1} = \frac{t}{C_\alpha(i, t-1|M) + 1}. \tag{2}$$

The multiplier comes from that each subject choice values as 1 with the total points $C_A + C_B + 1 = t$ being divided among $C_\alpha + 1$ subjects who have chosen α. This is similar to the payoff odds of the parimutuel system in gambling. Figure 1 displays the experimental design.

In EXP-II, in addition to the three cases $r \in \{O, C, M\}$, the subjects answered in at most four cases $r \in \{1, 5, 11, 21\}$ between cases O and C. In case $r \in \{1, 5, 11, 21\}$, the subject received summary statistics $\{C_A(i, t-1|r), C_A(i, t-1|r)\}$ from his previous r subjects. $C_A(i, t-1|r) + C_B(i, t-1|r) = r$ holds and as r increases, the amount of social information increases. In EXP-I, the amount of social information increases rapidly from $r = 0$ in case O to $r = t-1$ in case C. In EXP-II, r gradually increases. The payoff for the correct choice is 1 in case $r \in \{O, 1, 5, 11, 21, C\}$ and the multiplier in case M. Detailed information about EXP-II has been presented in our previous work [16], where we have studied the experimental data for cases $r \in \{O, 1, 5, 11, 21, C\}$. In this paper, we concentrate on cases $r \in \{O, C, M\}$.

There were two groups (A and B) of subjects and we repeated the same experiment. We obtained 120×2 sequences $X(i,t|r)$ for each $r \in \{O, C, M\}$. We label the sequence in group B by $i + 120$, so that $i \in \{1, 2, \cdots, 240\}$. We denote the length of sequence $\{X(i,t|r)\}$ by T_i.

TABLE I. Experimental design. T means the number of subjects and $\{r\}$ means the cases where the subjects answered the quiz. I means the number of questions.

Experiment	Group	T	$\{r\}$	I
EXP-I	A	57	$\{O, C, M\}$	120
EXP-I	B	63	$\{O, C, M\}$	120
EXP-II	A	52	$\{O, 1, 5, 11, 21, C, M\}$	120
EXP-II	B	52	$\{O, 1, 5, 11, 21, C, M\}$	120

B. Experimental procedure

FIG. 2. Snapshot of the screen for cases O, C and M. Summary statistics $\{C_A, C_B\}$ (multipliers $\{M_A, M_B\}$) are given in the second row in the box in case $C(M)$.

We explain the experimental procedure in EXP-I in detail. There were five sessions for each group. In one session, 10 to 13 subjects entered the laboratory and sat in the

partitioned spaces. After listening to a brief explanation about the experiment and the reward, in particular about the multiplier, they logged into the experiment web site using their IDs and started to answer the questions. Interaction between subjects was permitted only through the social information given by the experiment server. A question was chosen by the experiment server and displayed on the monitor. First, subjects answered the fist half of the 120 questions $i \in \{1, 2, \cdots, 60\}$ using their own knowledge only ($r = O$). If a subject answers after t subjects, it is denoted as $X(i, t+1|O)$. After answering all the sixty questions in case O, the subjects answered the same 60 questions in case C. For question i, the order t' that the subject answer in case C is in general different from the order t in case O (see Figure 1). Finally, the subjects answered the same questions in case M. The order t'' of the subject to question i is in general different from t and t' in the previous two cases. In each case, the experiment server chose a question among the sixty questions at random that was not served to the other subjects. After a five-minute interval, we repeated the same procedure so that the subjects answered all 120 questions. By performing five sessions, we have gathered data for 57 (63) subjects in group A (B)

Figure 2 shows the experience of the subjects. In the example covered in the figure, already nine subjects have answered question 30. In case O, no social information is given. The subject chooses among the two options. In case C, the summary statistics from the previous nine subjects' choices are displayed in the second row of the box. Taking into account this information, the subject makes a choice. In case M, the multipliers are given in the second row along with the number of subjects who answered the question. Only one subject among nine has chosen A and remaining eight subjects have chosen B. Multiplier M_A (M_B) is calculated as $10/(1+1) = 5$ ($10/(8+1) = 1.1$). The multipliers are rounded off to one decimal place. In EXP-II, the experiences of the subjects are almost the same as those of the subjects in EXP-I [16]. The difference lies in the subjects answering each question from $r = O$ to $r = M$ before proceeding to another question. Accordingly, $t = t' = t''$ holds. In EXP-II, the subjects were likely to easily remember the answers for the earlier cases with less social information and be careful in choosing answers in the later cases with more social information. In order to exclude such an effect, we changed the system to that in EXP-I.

C. Max-Min Strategy in case M

We discuss what is the optimized strategy for herders in case M. In the experiment, a subject can choose $\alpha \in \{A, B\}$. We suppose that he/she votes one unit for a choice and call him/her a voter. Here, we consider the case where one vote can be divided by the voter. If a voter believes A is correct, he/she votes one unit for A. If a voter does not know the answer at all, he/she votes 0.5 unit for A and 0.5 unit for B. The multiplier for choice $\alpha \in \{A, B\}$ is M_α. We assume that a voter thinks the probability that A is correct is β, and the probability that B is correct is $1-\beta$. The voter divides one unit vote into x for A and $1-x$ for B by his/her decision making. Expected return R is

$$\begin{aligned} R &= \beta \cdot M_A \cdot x + (1-\beta) \cdot M_B \cdot (1-x) \\ &= \beta(M_A x - M_B(1-x)) + M_B(1-x). \end{aligned} \tag{3}$$

We assume that herders do not have information about the correct answers without multiplier $\{M_\alpha\}, \alpha \in \{A, B\}$. Hence, we assume that a herder cannot estimate the probabilities of correct answers β as Knightian uncertainty, because a herder has no knowledge to an-

swer the question [24]. In this circumstance, the max-min strategy is proved to be optimal in game theory [21]. The voter minimizes the expected loss due to the uncertainty in the choice. In order to minimize the expected loss from the uncertainty, it should be chosen so that $M_A \cdot x = M_B \cdot (1-x)$ holds, from (3). This position has no sensibility for β.

We can calculate x from (3)

$$x = \frac{M_B}{M_B + M_A}. \tag{4}$$

As multiplier M_α is calculated as

$$M_\alpha = \frac{t+1}{C_\alpha + 1},$$

ratio x for A is then

$$x = \frac{C_A + 1}{t+2} \sim \frac{C_A}{t} \quad \text{for} \quad t >> 1. \tag{5}$$

x becomes proportional to C_A and it is the voting strategy of analog herders.

In our experiment, it is not possible to realize the optimal mixed-strategy at the individual level, because the voter cannot divide one's vote (choice). It can be realized only collectively. Hence, the averaged behavior of herders becomes akin to that of the analog herders, when herders adopt the optimized strategy.

III. DATA ANALYSIS : MACROSCOPIC ASPECTS

We obtained 240 sequences $\{X(i,t|r)\}, t \in \{1, 2, \cdots, T_i\}$ for question $i \in \{1, \cdots, 240\}$ and case $r \in \{O, C, M\}$ in each experiment [25]. The percentage of correct answers of sequence $\{X(i,t|r)\}$ for question i in case r is defined as $Z(i|r) = \sum_{s=1}^{T_i} X(i,s|r)/T_i$. In the analysis, the subjects are classified into two categories – independent and herder – for each question. We assume that the probability of correct choice for independent and herder subjects is 100% and 50%, respectively [16]. For a group with $p(i)$ herder and $1-p(i)$ independent subjects, the expectation value of $Z(i|O)$ is $1-p(i)/2$. The maximal likelihood estimate of $p(i)$ is given as $p(i) = 2(1-Z(i|O))$. The assumption of the random guess (50%) by the herder might be too simple. As $Z(i|O)$ approaches 0.5 and almost all subjects do not know the answer to the question, $p(i)$ approaches 100% and the estimate works well.

A. Distribution of $Z(i|r)$

There are 240 samples of sequences of choices for each r. We divide these samples into 11 bins according to the size of $Z(i|r)$, as shown in Table II. The samples in each bin of case O share almost the same value of p. For example, in the samples of No. 6 bin ($0.45 < Z(i|O) \leq 0.55$), there are almost only herders in the subjects' sequence and $p(i) \simeq 100\%$. In contrast, in the samples of No. 11 bin ($Z(i|O) > 0.95$), almost all subjects know the answer to the questions and are independent ($p(i) \simeq 0\%$). An extremely small value of $Z(i|O)$ indicates some bias in the question. In addition, the minimum value of $Z(i|r)$ should be $1-p(i)$. We omit the samples that satisfy $Z(i|O) < 0.45$ or $Z(i|C) < 1-p(i)$ or $Z(i|M) < 1-p(i)$. By these procedures, we are left with 167 (177) samples in EXP-I (II) and we denote the set by I'. I(No.) denotes the set of samples in each bin in case O among I'. The samples with $Z(i|O) < 0.5$ in $I(6)$ have $p(i)$ values larger than 100%. These

TABLE II. Effect of social information on subjects' decisions. We divide the samples according to the size of $Z(i|r)$. $N(\text{No.}|r)$ denotes the number of samples for case r in each bin. $I(\text{No.})$ is the set of sample i in each bin of case O after removing the samples that satisfy $Z(i|O) < 45\%$ or $Z(i|C) < (1-p(i))$ or $Z(i|M) < (1-p(i))$. We use the same notation, $I(\text{No.})$, for the number of samples in the set. p_{avg} is estimated as the average value of $p(i) = 2(1-Z(i|O))$ over the samples in $I(\text{No.})$. In the last two columns, the sub-optimal ratios for the samples of $I(\text{No.})$ in case $r \in \{C, M\}$ are shown.

EXP-I

No.	$Z(i\|r)$[%]	$N(\text{No.}\|O)$	$N(\text{No.}\|C)$	$N(\text{No.}\|M)$	$I(\text{No.})$	$p_{avg}(\text{No.})$[%]	$Z(i\|C) < 1/2$	$Z(i\|M) < 1/2$
1	< 5	0	5	0	NA	NA	NA	NA
2	$5 \sim 15$	3	33	7	NA	NA	NA	NA
3	$15 \sim 25$	5	28	25	NA	NA	NA	NA
4	$25 \sim 35$	18	9	30	NA	NA	NA	NA
5	$35 \sim 45$	35	5	13	NA	NA	NA	NA
6	$45 \sim 55$	38	5	13	38	97.5	18/38	17/38
7	$55 \sim 65$	57	5	14	52	78.3	7/52	5/52
8	$65 \sim 75$	29	7	19	26	60.3	0/26	0/26
9	$75 \sim 85$	41	17	44	38	40.6	0/38	0/38
10	$85 \sim 95$	11	57	62	11	21.3	0/11	0/11
11	≥ 95	3	69	13	2	5.1	0/2	0/2
Total		240	240	240	167	66.8%	35/167	22/167

EXP-II

No.	$Z(i\|r)$[%]	$N(\text{No.}\|O)$	$N(\text{No.}\|C)$	$N(\text{No.}\|M)$	$I(\text{No.})$	$p_{avg}(\text{No.})$[%]	$Z(i\|C) < 1/2$	$Z(i\|M) < 1/2$
1	< 5	0	2	0	NA	NA	NA	NA
2	$5 \sim 15$	0	18	6	NA	NA	NA	NA
3	$15 \sim 25$	8	22	18	NA	NA	NA	NA
4	$25 \sim 35$	16	20	23	NA	NA	NA	NA
5	$35 \sim 45$	36	8	16	NA	NA	NA	NA
6	$45 \sim 55$	43	9	19	43	96.7	16/43	15/43
7	$55 \sim 65$	46	10	16	45	79.3	8/45	3/45
8	$65 \sim 75$	45	14	26	45	62.7	2/45	0/45
9	$75 \sim 85$	33	33	56	33	41.9	0/33	0/33
10	$85 \sim 95$	11	67	54	11	21.3	0/11	0/11
11	≥ 95	2	37	6	0	NA	NA	NA
Total		240	240	240	177	68.7%	26/177	18/177

values are errors of the estimation $p(i) = 2(1-Z(i|O))$. The standard deviation of $Z(i|O)$ is $\sqrt{p(i)/4T_i}$. In the estimation of $p(i)$, there is fluctuation with the magnitude of $\sqrt{p(i)/T_i}$. If $p(i)$ takes a value larger than 100%, we take it to be 100%.

Table II shows the number of data samples in each bin for case $r \in \{O, C, M\}$ as $N(\text{No.}|r)$. Social information causes remarkable changes in subjects' choices. For case O, there is one peak at No. 7, and for case $C(M)$, there are peaks at No. 2 (4) and No. 11 (10) in EXP-I. We calculate the average value of $p(i)$ for the samples in $I(\text{No.}|O)$. We denote it as $p_{avg}(\text{No.})$

and estimate it as

$$p_{avg}(\text{No.}) = \frac{1}{I(\text{No.})} \sum_{i \in I(\text{No.})} p(i).$$

Here, $I(\text{No.})$ in the denominator means the number of samples in $I(\text{No.})$, which is given in the sixth column of the table. We show the results in the seventh column. In the last two columns, we show the ratio of sub-optimal cases $\{Z(i|r) < 1/2\}$ for $r \in \{C, M\}$ among the samples in $I(\text{No.}|O)$. In both cases, as p_{avg} increases, the sub-optimal ratio increases rapidly to about half.

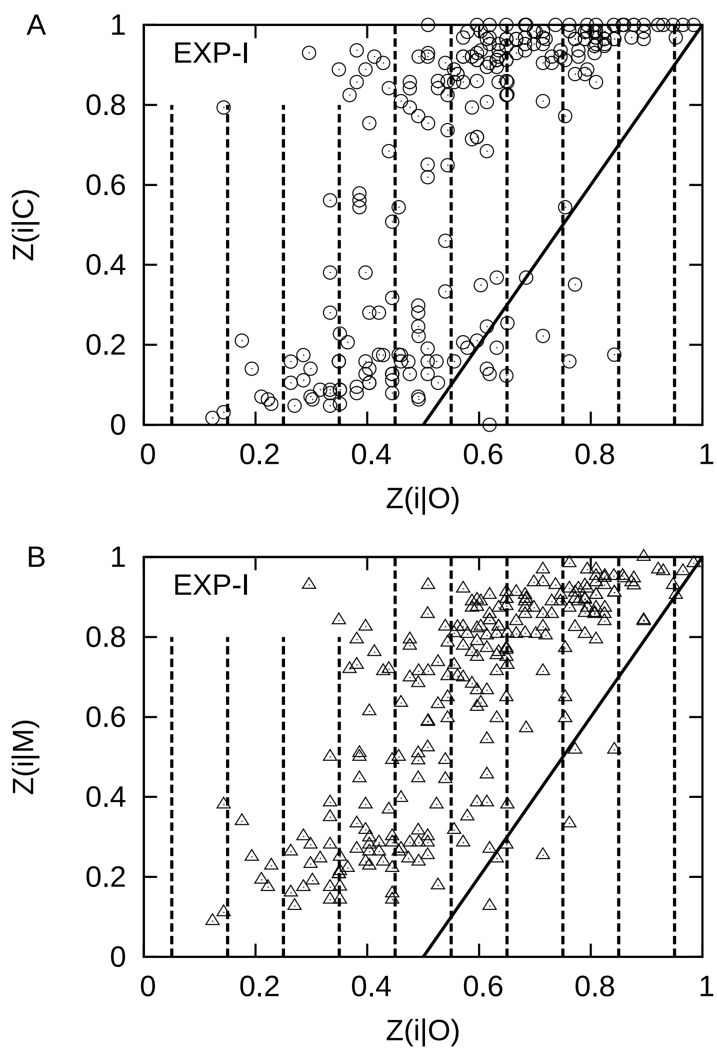

FIG. 3. Scatter plots of $Z(i,T_i|O)$ vs $Z(i,T_i|r)$ for (A) Case C and (B) Case M. The vertical lines show the border of the bins in Table II. The rising diagonal line from $(0.5, 0)$ to top right shows the boundary condition $Z(i|r) = 1 - p$.

In order to see the social influence more pictorially, we show the scatter plots of $Z(i|O)$ vs $Z(i|r), r \in \{C, M\}$ of EXP-I in Fig. 3. The x-axis shows $Z(i|O)$ and the y-axis shows $Z(i|r)$. The vertical lines show the boundary between the bins (from No. 1 to No. 11) for case O in Table II. The rising diagonal line from (0.5.0) to top right shows the boundary condition

$Z(i|r) = 1 - p$. If subjects' answers are not affected by social information, data would distribute on the diagonal line from $(0, 0)$ to top right. As the plots clearly indicate, the samples scatter more widely in the plane in case C than in case M, which means that social influence is bigger in case C. For the samples with $Z(i|O) \geq 0.65$ in case O (Nos. 8, 9, 10 and 11 bins in Table II), the changes, $Z(i|C)-Z(i|O)$, are almost positive and $Z(i|C)$ takes a value of about 1 in case C. In case M, the changes, $Z(i|M)-Z(i|O)$, are also almost positive and $Z(i|M)$ takes a value of about 0.9. Average performance improves by social information for the samples in both cases. In contrast, for the samples with $0.45 \leq Z(i|O) < 0.65$ (Nos. 6 and 7 bins in Table II), social information does not necessarily improve average performance. There are many samples with $Z(i|r) - Z(i|O) < 0$ in both cases. These samples are in the sub-optimal state and constitute the lower peak in Table II.

B. Asymptotic behavior of the convergence

We have seen drastic changes in the distribution of $Z(i|r)$ from the distribution of $Z(i|O)$. Table II and Figure 3 show the two-peak structure in the distribution of $Z(i|r)$. In our previous work on the information cascade phase transition [16], we have studied the time dependence of the convergence behavior of the sequences $\{X(i,t|r)\}$. We denote the ratio of correct answers, $\frac{C_1(i,t|r)}{t}$, as

$$Z(i,t|r) = \frac{C_1(i,t|r)}{t} = \frac{1}{t}\sum_{s=1}^{t} X(i,s|r). \tag{6}$$

$Z(i,T_i|r) = Z(i|r)$ holds by definition. By studying the asymptotic behavior of the convergence of sequence $\{Z(i,t|r)\}$ for the samples in $I(\text{No.})$, one can clarify the possibility of the information cascade transition by varying p. The variance of $Z(i,t|r)$ for the samples in $I(\text{No.})$ is defined as

$$\begin{aligned}
&\text{Var}(Z(i,t|r))_{\text{No.}} \\
&= \frac{1}{I(\text{No.})}\sum_{i\in I(\text{No.})} (Z(i,t|r)- < Z(i,t|r) >_{\text{No.}})^2 \\
&< Z(i,t|r) >_{\text{No.}} = \frac{1}{I(\text{No.})}\sum_{i\in I(\text{No.})} Z(i,t|r).
\end{aligned} \tag{7}$$

Here, we denote the average value of $Z(i,t|r)$ over the samples in $I(\text{No.})$ by $< Z(i,t|r) >_{\text{No.}}$. In the one-peak phase, $\text{Var}(Z(i,t|r))$ converges to zero in thermodynamic limit $t \to \infty$. Depending on the convergence behavior, the one-peak phase is classified into two phases [18]. If the variance of $Z(i,t|r)$ shows normal diffusive behavior as $\text{Var}((Z(i,t|r)) \propto t^{-1}$, it is called the normal diffusion phase. We note that the variance is estimated for the ratio, $C_1(i,t|r)/t$, and the usual behavior t^1 for the sum of t random variables is replaced by $\propto t/t^2 = t^{-1}$. If convergence is slow and $\text{Var}(Z(i,t|r)) \propto t^{-\gamma}$ with $0 < \gamma < 1$, it is called the super diffusion phase [26]. In the two-peak phase, $\text{Var}(Z(i,t|r))$ converges to some finite value in limit $t \to \infty$ [17].

Figure 4 shows the double logarithmic plots of $\text{Var}(Z(i,t|r))_{\text{No.}}$ as a function of t. If the plot has a negative slope $-\gamma$ with $\gamma > 0$ in limit $t \to \infty$, the system is in the one-peak phase.

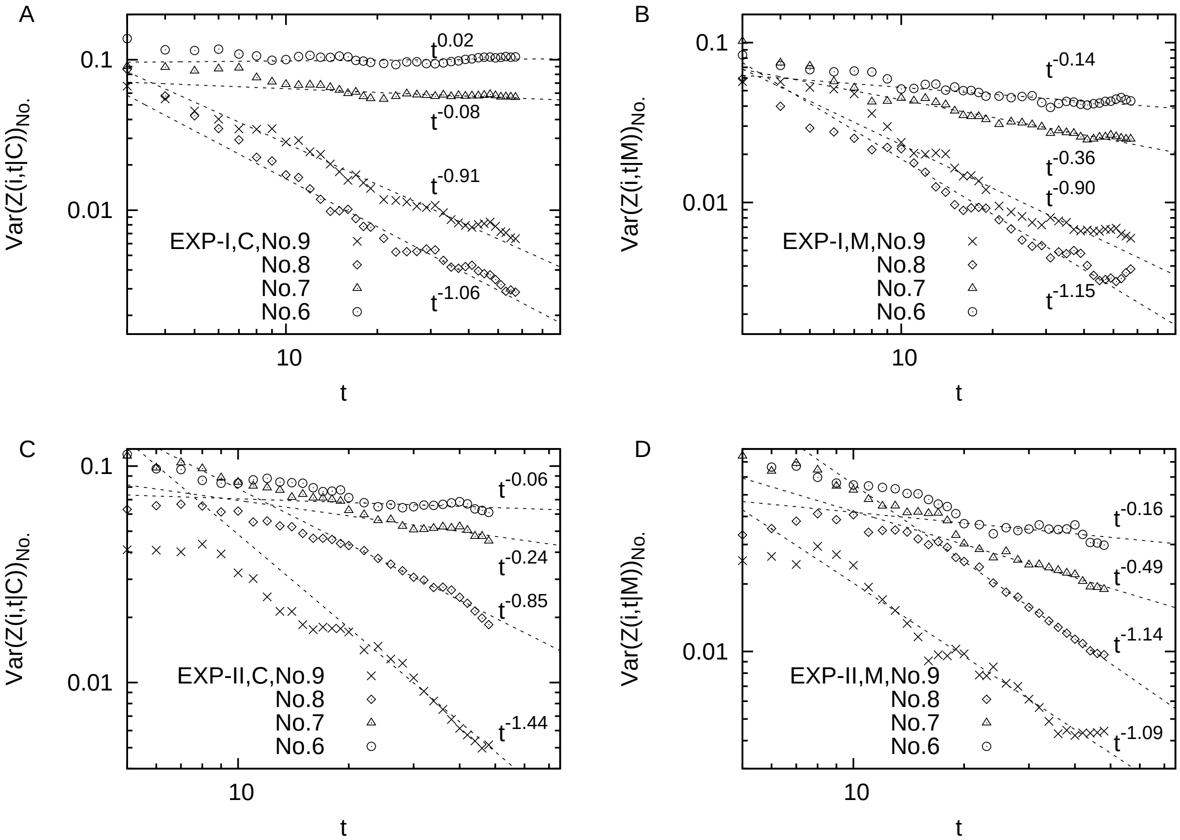

FIG. 4. Convergent behavior. Cconvergence is given by the double logarithmic plot of $\mathrm{Var}(Z(i,t|r))_{\mathrm{No.}}$ vs t using the samples in the four bins (No. 6 ($\circ$), 7 ($\triangle$), 8 ($\diamond$) and 9 ($\times$) in Table II) for (A) Case C in EXP-I, (B) Case M in EXP-I, (C) Case C in EXP-II, and (D) Case M in EXP-II. The dotted lines are fitted results with $\propto t^{-\gamma}$ for $t \geq 10(20)$ in EXP-I (II).

If slope is zero and $\gamma = 0$ in the limit, the system is in the two-peak phase. We see that convergence becomes very slow as $p_{avg}(No.)$ increases in general. Exponent γ is estimated by fitting with $\propto t^{-\gamma}$ for $t \geq 10$ in EXP-I. It decreases from almost 1 to $-0.02(0.14)$ with an increase in p_{avg} in case C (M). For the samples in $I(9)$ and $I(8)$, γs are almost 1 and the system is in the normal diffusion (one-peak) phase in both cases $r \in \{C, M\}$. For the samples in $I(7)$, γs are apparently smaller than 1 and the system might be in the super diffusion phase. However, system size T is very limited and in thermodynamic limit $T \to \infty$, γs might converge to 1. For the samples in $I(6)$, γ becomes negative ($\gamma = -0.02$) in case C. This suggests that the system is in the two-peak phase for the samples in $I(6)$ and threshold value p_c is between 78.5% and 97.5%. In case M, γ is negative even for the samples in $I(6)$ and the system might be in the super diffusion phase. However, the result does not necessarily deny the existence of the two-peak phase, taking into account the estimate error of $p(i)$ and the estimate error of γ from the limited sample size. We can only say that if the two-peak phase exists, threshold value p_c in case M is considerably larger that that in case C.

IV. DATA ANALYSIS: MICROSCOPIC ASPECTS

In this section, we study the microscopic aspects of herders. We clarify how they copy others' choices and derive a microscopic rule in each case $r \in \{C, M\}$. In particular, we study whether they behave as an analog herder like in case M.

A. How do herders copy others?

We determine how a herder's decision depends on social information. For this purpose, we need to subtract independent subjects' contribution from $X(i, t+1|r)$. The probability of being independent is $1 - p(i)$, and such a subject always chooses 1. A herder's contribution is estimated as

$$(X(i, t+1|r) - (1 - p(i)))/p(i).$$

How the herder's decision depends on $C_1(i, t|r) = n_1$ is estimated by the expectation value of $(X(i, t+1|r) - (1 - p(i))/p(i)$ under this condition. The expectation value means the probability that a herder chooses an option under the influence of the prior n_1 subjects among t who choose the same option. We denote it by $q_h(t, n_1|r)$, and estimate it as

$$q_h(t, n_1|r) = \frac{\sum_{i \in I'} \left[\frac{X(i,t+1|r)-(1-p(i))}{p(i)}\right] \delta_{C_1(i,t|r),n_1}}{\sum_{i \in I'} \delta_{C_1(i,t|r),n_1}}. \tag{8}$$

Here, $\delta_{i,j}$ is 1 (0) if $i = j$ $(i \neq j)$ and the denominator is the number of sequences where $C_1(i, t|r) = n_1$. From the symmetry between $1 \leftrightarrow 0$, we assume that $q_h(t, n_1) = 1 - q_h(t, t - n_1)$. We study the dependence of $q_h(t, n_1)$ on n_1/t and round n_1/t to the nearest values in $\{k/13(12)|k \in \{0, 1, 2, \cdots, 13(12)\}\}$ in EXP-I (II).

Figure 5 shows the plot of $q_h(t, n_1|r)$ for (A) case C and (B) case M. We can clearly see the strong tendency to copy others in case C. As n_1/t increases from $1/2$, $q_h(t, n_1|C)$ rapidly increases and the slope at $n_1/t = 1/2$ is about 2.0 in EXP-I. Such nonlinear behavior is known as quorum response in social science and ethology [27]. The magnitude of the slope measures the strength of herders'response. Comparing EXP-I and EXP-II, the response of the herder is more sharp in EXP-I than in EXP-II. In EXP-II, where the amount of social information increases gradually, the subjects tend to copy others' choices more prudent than in EXP-I. If the slope exceeds 1, the system shows the information cascade phase transition. Transition ratio p_c depends on the slope. In the digital herder case, where $q_h(t, n_1) = \theta(n_1 - t/2)$ and the slope is infinite, p_c takes 0.5 [17]. As the slope reduces to 1, p_c increases to 1 and the phase transition disappears in the limit [23].

Contrary to case C, the dependence of $q_h(t, n_1|M)$ on n_1/t is weak and the slope at $n_1/t = 1/2$ is almost 1 in case M. In range $1/4 \leq n_1/t \leq 3/4$, $q_h(t, n_1|M)$ lies on the diagonal dotted line and the herders almost behave as analog herders. They collectively adopted the optimal mixed-strategy. As the slope at $n_1/t = 1/2$ is small, if the information cascade phase transition occurs, transition ratio p_c should become large as compared to in case C. The wrong convergence of the majority choice occurs only when almost nobody knows the correct answer in case M. One can also see an interesting behavior of herders. If minority choice ratio n_1/t is smaller than $1/4$ and multiplier m exceeds 4, some herders make the choice. As a result, if $n_1/t > 3/4$, $q_h(t, n_1|M)$ becomes almost constant, about $3/4$. We can interpret this as some of the herders prefering big multiplier (long-shot) and

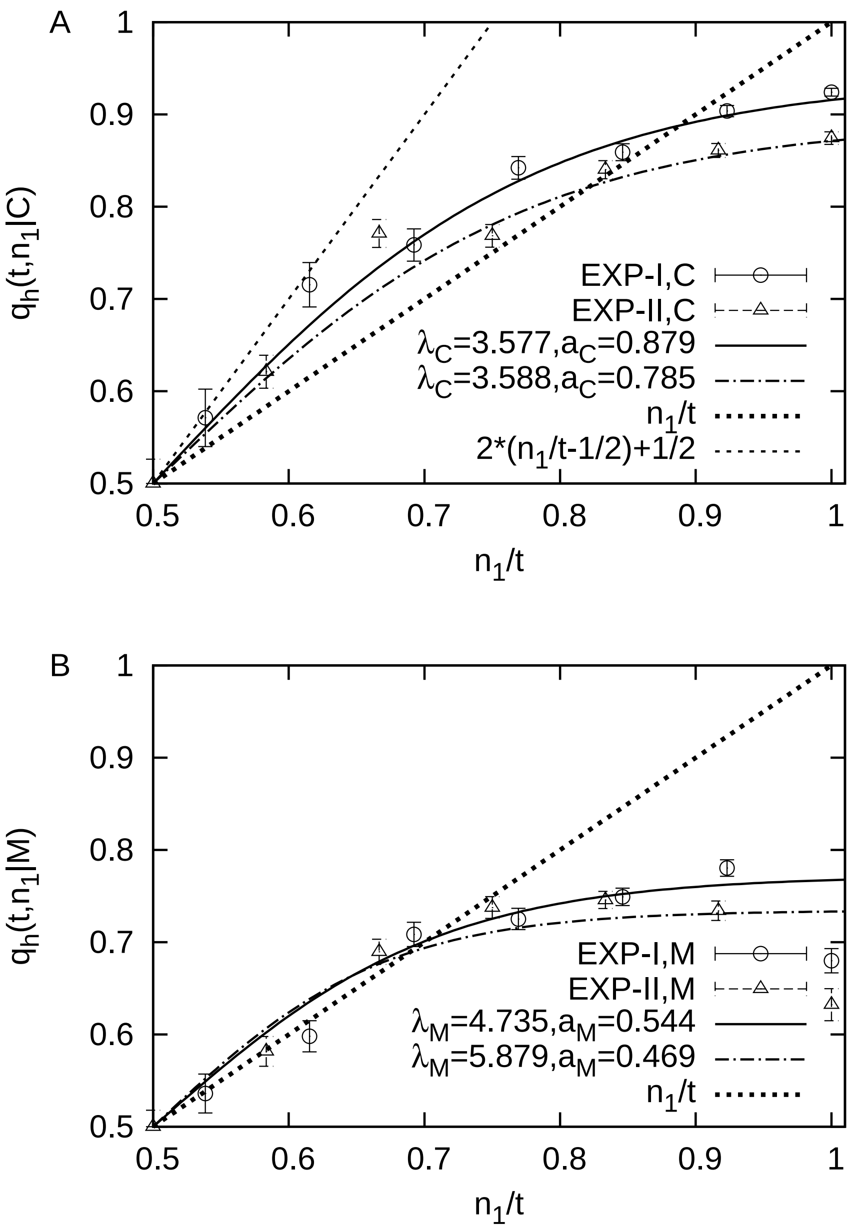

FIG. 5. Microscopic rule of herder's decision for (A) Case C and (B) Case M. It shows the probability $q_h(t, n_1|r)$ that a herder chooses an option under the influence of the prior n_1 subjects among t choosing it in case r. The solid curves are fitted results with Eq.(9). The dotted diagonal line shows the analog herder model $q_h(t, n_1) = n_1/t$. The thin dashed line in (A) shows $2(n_1/t - 1/2) + 1/2$.

$q_h(t, n_1)$ saturating at 3/4. This behavior is not rational and is known as favorite-longshot bias in race-track betting markets [19].

B. Average herders and logistic herders

As in our previous work [16], we model the behavior of herders by the following functional form:

$$q_h(t, n_1|r) = \frac{1}{2}\left(a_r \tanh(\lambda_r(n_1/t - 1/2)) + 1\right). \tag{9}$$

Parameters a_r and λ_r indicate the strength of the conformity of subjects. a_r is the net ratio of herders who react positively to prior subjects' choices [28]. λ_r denotes the strength of the response on social information. Combined factor $\frac{1}{2}a_r\lambda_r$ is the slope of the functional form at $n_1/t = 1/2$. We call the herder who chooses according to eq.(9) a logistic herder, and this model as the logistic herder (LH) model. In addition, we introduce an average herder (AvH) model where $q_h(t, n_1|r)$ is given by linear extraporation of the values in eq.(8). We describe the estimation procedure of parameters $\{\lambda_r, a_r\}$ of the LH model in Appendix C. The estimation results of the parameters, $\log L$ (eq.C4) and AIC (eq.(C5)), for the LH model are given in Table III. The fitted results are shown in Fig. 5. For comparison, we show the log likelihood $\log L$ and AIC of the AvH model. The AvH model has seven (six) parameters as $k \in \{7, 8, 9, \cdots, 13\ (12)\}$ in EXP-I (II).

TABLE III. Parameter estimates for the logistic herder (LH) model. We show the fitted results, λ_r, a_r, of the LH model for case $r \in \{C, M\}$ in EXP-I and EXP-II. The fourth column shows the number of observations and the fifth column shows the number of parameters N_p of the model. In the eighth column, $\log L$ is given. The last column shows AIC. For comparison, we show $\log L$ and AIC for the average herder (AvH) model.

EXP	r	Model	#Obs	N_p	λ_r	a_r	$\log L$	AIC
I	C	AvH	9850	7	NA	NA	-2951.6	5917.1
I	C	LH	9850	2	3.58	0.879	-2951.5	5907.1
I	M	AvH	9819	7	NA	NA	-4592.5	9198.9
I	M	LH	9819	2	4.73	0.544	-4601.6	9207.2
II	C	AvH	8809	6	NA	NA	-3468.8	6949.5
II	C	LH	8809	2	3.59	0.785	-3470.6	6945.3
II	M	AvH	8809	6	NA	NA	-4406.6	8825.3
II	M	LH	8809	2	5.88	0.469	-4418.3	8840.5

In case C, $\log L$ of the LH model is comparable to that of the AvH model. The LH model describes the behavior of the herders as well as the AvH model. As N_P of the LH model is smaller than that of the AvH mode, the LH model is better than the AvH model from the viewpoint of AIC. The slope at $n_1/t = 1/2$ in the LH model is estimated as $\frac{1}{2}a_C\lambda_C = 1.57(1.41)$ in EXP-I (II). In contrast, in the AvH model, the slope is roughly 2 in EXP-I (see Figure 5A). As the slopes are larger than 1, the information cascade phase transition occurs at $p_c < 100\%$. As the value of the slope is crucial in the estimation of system performance, we study $q_h(t, n_1|C)$ at $n_1/t = 1/2$ in detail in the next subsection. We defer which model to adopt in case C to the place.

In case M, the LH model does not describe herders' behavior well. Comparing $\log L$ of the two models, we see that the AvH model is more faithful to herders' behavior. A comparison of AIC yields that the AvH model is better than the LH model. As one can see clearly from Figure 5B, the slope at $n_1/t = 1/2$ is almost 1, and at $n_1/t = 0.7$, $q_h(t, n_1|M)$ passes the diagonal line. It looks linear or downward convex in region $1/2 \leq n_1/t \leq 3/4$. In

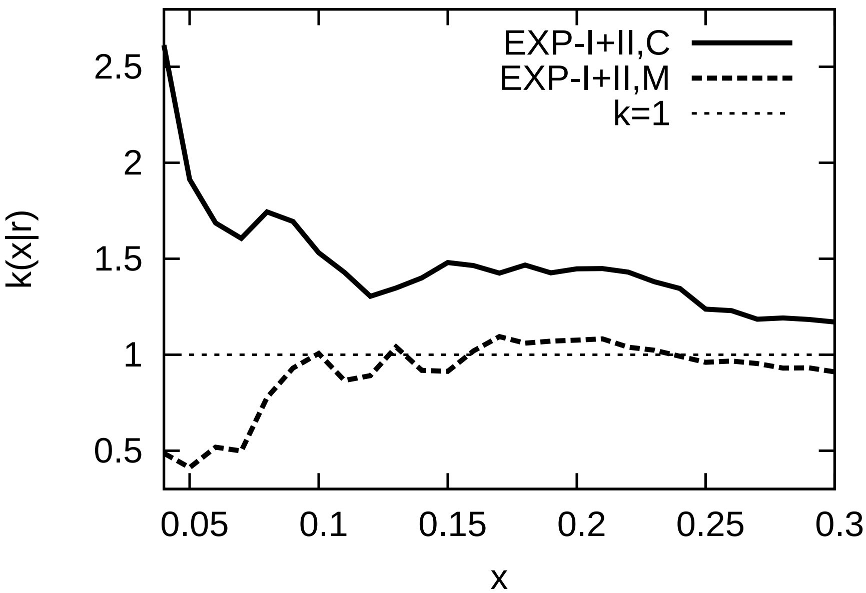

FIG. 6. Plots of $k(x|r)$ vs x for $r \in \{C, M\}$. We use all data $\{X(i,t|r)\}$ from EXP-I and EXP-II that satisfy $|C_1(i,t-1|r)/(t-1) - 1/2| < x$, and fit k in $q_h(t,n_1|r) = k(n_1/t - 1/2) + 1/2$ by the maximum likelihood estimate. The thick solid (dashed) line plots the results for case C (M). $k = 1$ (thin dotted line) corresponds to analog herder $q_h(t,n_1) = 1 \cdot n_1/t$.

contrast, the LH model is upward convex for $n_1/t > 1/2$, and the fitted result suggests that the slope at $n_1/t = 1/2$ is about 1.28 (1.38), by the estimation of $\frac{1}{2}a_M\lambda_M$ in EXP-I (II). These discrepancies are crucial and we adopt the AvH model to describe herders' behavior.

C. Behavior of $q_h(t,n_1|r)$ near $n_1/t = 1/2$

In case C, the LH model and the AvH model are equally faithful to describe the behavior of herders as they have almost the same $\log L$. There are discrepancies in the slope of $q_h(t,n_1|C)$ at $n_1/t = 1/2$. The magnitude of the slope is crucial to the properties and performance of the system. However, from the cascade effect, the samples move to the edges of $[0,1]$ on the n_1/t axis. In region $n_1/t \simeq 1/2$, the number of observations is small and the maximum likelihood estimate in the previous subsection does not capture the behavior near $n_1/t = 1/2$. We study $q_h(t,n_1|r)$ in the region in detail. As we are interested in the slope of $q_h(t,n_1|r)$ at $n_1/t = 1/2$, we assume the following functional form with slope parameter k,

$$q_h(t,n_1|r) = k \cdot (n_1/t - 1/2) + \frac{1}{2}.$$

Here, k is the slope at $n_1/t = 1/2$. We consider a region $[1/2 - x, 1/2 + x]$ with a small and positive parameter $x \leq 0.3$. We fit k by the maximum likelihood method and write the result as $k(x|r)$. In the estimation of l in eq.(C3), we use only the data $\{X(i,t|r)\}$ that satisfy $|C_1(i,t-1|r)/(t-1) - 1/2| < x$. We combine the data from EXP-I and EXP-II to address the data scarcity for small x.

Figure 6 plots $k(x|r)$ vs x for $r \in \{C, M\}$. We are interested in the slope at $n_1/t = 1/2$

and it is necessary to see limit $x \to 0$. As x decreases, the number of observations decreases and the estimation error increases. We set $x \geq 0.04$. In case C, $k(x|r)$ increases almost monotonically as x decreases. It takes about 2 at $x = 0.05$ and coincides with the behavior of $q_h(t, n_1|C)$ in Figure 5A. The AvH model describes herders' behavior of herder for $x \geq 0.05$. We adopt the AvH model to describe herders' behavior in case C. In case M, $k(x|r)$ starts from about half at $x = 0.04$ and rapidly increases to about 1 at $x = 0.1$. In range $0.1 \leq x \leq 0.25$, $k(x|r)$ is almost 1 and near $x = 0.2$ it exceeds 1 slightly. These behaviors reflect that $q_h(t, n_1|M)$ is downward convex in range $0.5 \leq n_1/t < 0.55$ and upto $n_1/t = 3/4$, $q_h(t, n_1|M)$ almost behaves as n_1/t. As in case C, we adopt the AvH model to describe herders' behavior.

V. STOCHASTIC MODEL AND SIMULATION STUDY

The asymptotic analysis of the convergence of $Z(i, t|r)$ in case C shows the possibility of the two-peak phase for the samples in $I(6)$. Exponent γ is remarkably small, -0.02 (0.06) in EXP-I (II). In case M, the exponent of convergence for the same samples is negative, 0.14 (0.16), in EXP-I (II). However, the system sizes are limited and this causes the estimate error in $p(i)$. In addition, the estimate error from the very limited number of samples in each bin cannot be neglected. In this section, based on the herders' microscopic rule derived in the previous section, we simulate the system. We estimate the transition ratio $p_c(r)$ of the information cascade phase transition. In addition, we compare the performance of the system in case C and that in case M.

A. Voting model and transition ratio p_c

To understand the behavior of the system in thermodynamic limit $T \to \infty$, we simulate the system for large T by a stochastic model based on eq.(8). We introduce a stochastic process $\{X(t|r,p)\}, t \in \{1, 2, 3, \cdots, T\}$ for $r \in \{C, M\}$ and $p \in [0, 1]$. $X(t+1|r,p) \in \{0, 1\}$ is a Bernoulli random variable. Its probabilistic rule depends on $C_1(t) = \sum_{t'=1}^{t} X(t'|r,p)$ and the herders' proportion p. Given $\{C_1(t) = n_1\}$, the probabilistic rule that $X(t+1|r,p)$ obeys is

$$\begin{aligned} \text{Prob}(X(t+1|r,p) = 1|n_1) &= (1-p) + p \cdot q_h(t, n_1|r) \\ &\equiv q(t, n_1|r, p), \\ \text{Prob}(X(t+1|r,p) = 0|n_1) &= p \cdot (1 - q_h(t, n_1|r)). \end{aligned} \tag{10}$$

Here, we denote the probability that $X(t+1|r,p)$ takes 1 under the condition by $q(t, n_1|r, p)$. We adopt the AvH model for $q_h(t, n_1|r)$ and use the results of the experiments (Figure 5). We denote probability function $\text{Prob}(C_1(t) = n)$ for r and p as $P(t, n|r, p)$. The master equation for $P(t, n|r, p)$ is

$$\begin{aligned} P(t+1, n|r, p) &= q(t, n-1|r, p) \cdot P(t, n-1|r) \\ &\quad + (1 - q(t, n|r, p)) \cdot P(t, n|r). \end{aligned} \tag{11}$$

The expected value of $Z(t|r,p) = \frac{1}{t}C_1(t)$ is then estimated as

$$\mathrm{E}(Z(t|r,p)) = \sum_{n=0}^{t} P(t,n|r,p) \cdot \frac{n}{t}.$$

TABLE IV. Transition ratio p_c of the AvH model. We determine p_c by the condition that the self-consistent equation (eq.(12)) has multiple solutions for $p > p_c$.

EXP.	r	$p_c(r)$	r	$p_c(r)$
I	C	86.0%	M	95.7%
II	C	86.5%	M	96.7%

We are interested in the limit value of $Z(t|r,p)$ as $t \to \infty$, which we denote as z:

$$z \equiv \lim_{t\to\infty} Z(t|r,p).$$

In the one-peak phase, $Z(t|r,p)$ always converges to a value larger than half, which we denote as z_+. In the two-peak phase, in addition to z_+, $Z(t|r,p)$ converges to a value smaller than half, which we denote as z_-, with some positive probability. One cannot predict to which fixed point the system converges. It is a probabilistic process. To determine threshold value p_c between these phases and limit value $z_\pm$, one way is to solve the following self-consistent equation [18]:

$$z = q(t, t \cdot z|r,p). \tag{12}$$

Given p, if there is only one solution, it is z_+ and the system is in the one-peak phase. Convergence exponent γ is obtained by estimating the slope of $q(t, z \cdot t|r,p)$ at $z = z_+$ [18, 26]. If there are three solutions, which we denote as $z_1 < z_u < z_2$, z_1 (z_2) corresponds to z_- (z_+). Middle solution z_u is an unstable state and $Z(t|r,p)$ departs from z_u as t increases. The method gives the rigorous values for the LH model. In the AvH model, we think it works in case C where the self-consistent equation has at most three solutions (Figure 5A). In case M, the situation is uncertain. The self-consistent equation has five solutions at $p = 1.0$ and it is not clear whether the above theoretical analysis does work. With these notes in mind, we show p_c in Table IV. In case C, $p_c(C)$ is from 86.0% (EXP-I) to 86.5% (EXP-II). In case M, $p_c(M)$ is from 95.7% (EXP-I) to 96.7% (EXP-II).

In order to check the above results, we solve the master equation recursively and obtain $P(t,n|r,p)$ for $t \le T = 10^6$ for EXP-I. We estimate convergence exponent γ from the slope of $\mathrm{Var}(Z(t|r,p))$ as

$$\gamma = \log \frac{\mathrm{Var}(Z(T-\Delta T|r,p))}{\mathrm{Var}(Z(T|r,p))} / \log \frac{T}{T-\Delta T}. \tag{13}$$

For time horizons $T = 60$, we take $\Delta T = 50$ to match the analysis of the experimental data in Figures 4A and B. For $T = 10^3$ and 10^6, we take $\Delta T = 10^2$. In order to give the error bar of γ for the experimental results, we adopted the voting model to simulate the system and estimate the 95% confidence interval [16]. The results are summarized in Figure 7 for (A) case C and (B) case M. For $T = 60$, the model describes the experimental results well. γ shows non-monotonic behavior as a function of p, it is an artifact of a finite T. In limit $T \to \infty$, γ monotonically decreases from 1 to 0. For $T = 10^6$, γ becomes less than 10^{-2} at $p = 0.87$ (0.99) in case C (M) and gives the estimate of $p_c(r)$. These values are consistent

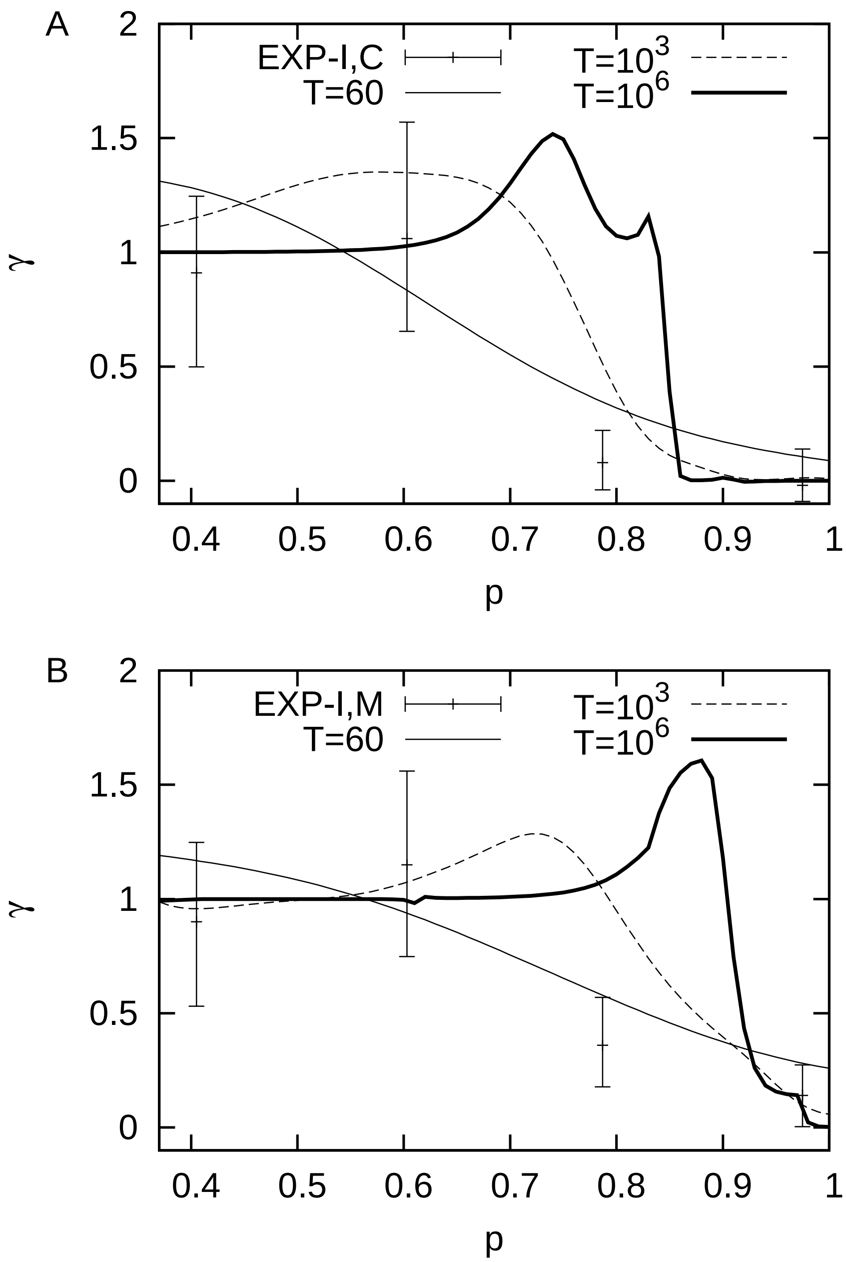

FIG. 7. Plot of γ vs p. We plot the results of the AvH model for EXP-I for (A) Case C and (B) for Case M. Symbol ($\circ$) denotes γs vs p_{avg} in EXP-I, which are estimated in Figure 4. The lines show the results of the stochastic model with system size $T = 60$ (thin solid), 10^3 (thin dashed), and 10^6 (thick solid).

with the ones given by the self-consistent equation in Table IV. For $p < p_c(r)$ ($p > p_c(r)$), the system in case $r \in \{C, M\}$ is in the one-peak (two-peak) phase.

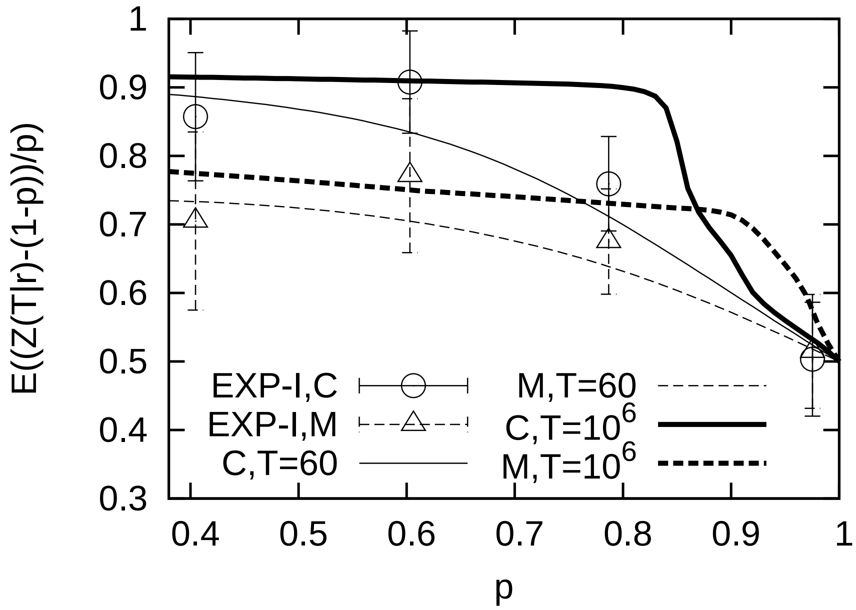

FIG. 8. Plot of herders' performance, $(\mathrm{E}(Z(T|r)) - (1-p))/p$ vs p for the voting model. Symbol $\circ$ ($\triangle$) indicates the experimental data for the four bins $I(6), I(7), I(8)$, and $I(9)$ in Table II for case C (M). The lines show the results of the stochastic model with system size $T = 60, r = C$ (thin solid), $T = 60, r = M$ (thin dashed), $10^6, r = C$ (thick solid), and $10^6, r = M$ (thick dashed).

B. Performance of herders

In order to compare the performances of herders in cases C and case M, we estimate the probability of choosing the correct answer by herders as a function of p [29]. As for the model, it can be estimated using the expectation value of $Z(t|r)$ as

$$\mathrm{E}(Z(t|r,p) - (1-p))/p).$$

For the experimental data, we take the average of $(Z(i|r) - (1-p(i)))/p(i)$ over the samples in $I(\text{No.})$:

$$\frac{1}{I(\text{No.})} \sum_{i \in I(\text{No.})} (Z(i|r) - (1-p(i)))/p(i).$$

We plot the results in Figure 8. The experimental results show that the performance of herders in case C is better than that in case M except for the samples in $I(6)$. As system size T increases, for $p < p_C(C)$, the performance in case C is better than that in case M. However, as p exceeds $p_c(C)$, the former rapidly decreases and dips below the latter. From the information cascade transition, herders' performance is much lowered and this results in the poor performance. In contrast, the poor performance of herders in case M for $p < p_C(C)$ comes from the saturation of $q_h(t, n_1|M)$ at about 3/4, the favorite-longshot bias. From the saturation, the performance of herders in case M cannot reach the high value. As a result, the performance in case M falls below that in case C for $p < p_C(C)$.

VI. CONCLUSIONS

Social influence, which here is restricted only to information regarding the choices of others, yields inaccuracy in the wisdom of crowd [2]. If a herder receives summary statistics $\{C_A, C_B\}$ and the payoff for the correct choice is constant, they strongy tends to copy the majority. The correct information given by independent voters are buried below the herd and the majority choice does not necessarily teach us the correct one if herders' proportion exceeds $p_c(C)$ [16]. By setting the return to be proportional to multipliers $\{M_A, M_B\}$ that are inversely proportional to summary statistics $\{C_A, C_B\}$, the situation changes drastically. In this case, the optimized behavior is that of an analog herder in game theory. An analog herder chooses $\alpha \in \{A, B\}$ with probability proportional to C_α. If a herder behaves as an analog herder, the phase transition to the two-peak phase can be avoided and the majority choice does converge to the correct one in the thermodynamic limit [23]. We studied herders' behavior under the influence of multipliers $\{M_A, M_B\}$ and showed that they behave almost as analog herders for $4/3 \leq m \leq 4$, where m is the multiplier. Outside the region, we see favorite-longshot bias [19], and observe that herders' copy probability $q_h(t, n_1|M)$ deviates from that of analog herders', $q_h(t, n_1) = n_1/t$. As a result, the threshold value p_c of the information cascade phase transition becomes extremely large.

The system size and number of samples in our experiment are very limited and it is difficult to estimate p_c precisely. In addition, in our experimental setup, the subjects have to choose between A and B. The optimized behavior can be adopted only collectively. An interesting problem is whether people can adopt the optimized behavior at the individual level or only collectively. In order to clarify this, we need to permit people to divide their choice and vote fractionally. If the fraction is proportional to the summary statistic of previous subjects' choices, it suggests that people can adopt the optimized behavior at the individual level. We think that more extensive experimental study of the system and of the related systems deserve further attention [30]. Such experimental studies should provide new approach to econophysics [22, 31–35] and socio-physics[36].

ACKNOWLEDGMENTS

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Appendix A: Additional information about Experiment

In EXP-I, 120 subjects were recruited from the Literature Department of Hokkaido University. In order to study the effect of social information on the choices of the subjects, it was necessary to control the transmission of information from others. We developed a web-based voting system by which multiple subjects could simultaneously participate in the experiment. The subjects used a web browser to access the web voting server in the intranet. Social information about others' choices was shown on the monitor.

Using slides, we showed subjects how the experiment would proceed. We explained that we were studying how their choices were affected by the choices of others. In particular, we emphasized that social information was realistic information calculated from the choices

of previous subjects. Through the slides, we also explained how to calculate multipliers $\{M_A, M_B\}$ in case M, with a concrete example. After the explanation, the experiment started. The subjects answered the 120 questions in the three cases within about one hour. Subjects were paid in cash upon being released from the session. There was a 500 yen (about 6 dollars) participation fee and additional rewards that were proportional to the number of points gained. In cases O and C, one correct choice was worth two points, and one point was worth one yen (about $1\frac{1}{4}$ cents). In case M, one correct choice was worth the multiplier itself. As for EXP-II, detailed information can be obtained from [16].

Appendix B: Quiz selection

We have used the same 120 questions in EXP-I and EXP-II. For the selection process, please refer to our previous paper [16]. Here, we study whether the difficulty of a question is an inherent property or not. For this purpose, we compare the percentage of correct answers to each question in case O in Group A and that in Group B. It is defined for Group A as $Z(i|O) = \sum_{s=1}^{T_i} X(i,s|O)/T_i$ and for Group B as $Z(i+120|O) = \sum_{s=1}^{T_{i+120}} X(i,s|O)/T_{i+120}$. We show the scatter plot $\{Z(i|O), Z(i+120|O)\}$ in Figure 9.

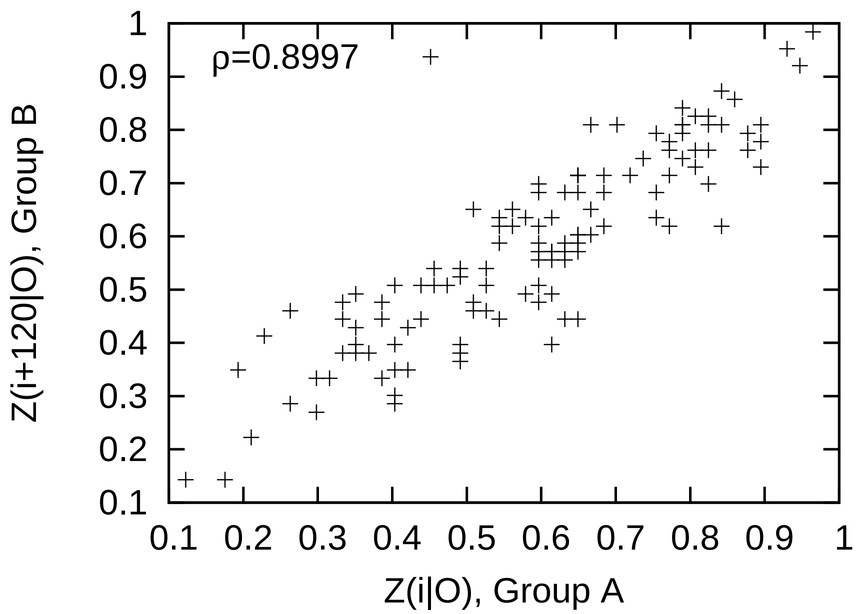

FIG. 9. Scatter plots of $Z(i|O)$ vs $Z(i+120|O)$ in EXP-I. Pearson's correlation coefficient ρ is 0.8997.

As one can clearly see the distribution almost on the diagonal line, we can infer that there is strong correlation. Pearson's correlation coefficient ρ is about 0.90. In EXP-II, we observe the same feature and ρ is about 0.82. The strong correlation means that if a question is difficult (easy) for the subjects in a group, it would also be difficult (easy) for the subjects in the other group. The system sizes in our experiments are very limited and there remains some fluctuation in the estimation of $Z(i|O)$, but it will disappear for a large system. We can control the difficulties of the questions in the experiment and study the

response of a subject under controlability. This aspect is important when one makes some prediction based on the results presented in this paper.

Appendix C: Estimation procedure of parameters $\{\lambda_r, a_r\}$ of the LH model

We describe the estimation procedure of parameters $\{\lambda_r, a_r\}$ of the LH model. We use standard maximum likelihood estimation [11]. Given $C_1(i,t|r) = n_1$, the probability that $X(i,t+1|r)$ takes 1 is

$$\begin{aligned}&\mathrm{Prob}(X(i,t+1|r)=1|C_1(i,t|r)=n_1)\\&=(1-p(i))\cdot 1+p(i)\cdot q_h(t,n_1|r).\end{aligned}\tag{C1}$$

We write the probability as

$$q(1|t,n_1,r)=\mathrm{Prob}(X(i,t+1|r)=1|C_1(i,t|r)=n_1).$$

The probability that $X(i,t+1|r)$ takes 0 under condition $C_1(i,t|r)=n_1$ is

$$\begin{aligned}&\mathrm{Prob}(X(i,t+1|r)=0|C_1(i,t|r)=n_1)\\&=p(i)\cdot(1-q_h(t,n_1))=1-q(1|t,n_1,r),\end{aligned}\tag{C2}$$

which we write as

$$q(0|t,n_1,r)=\mathrm{Prob}(X(i,t+1|r)=0|C_1(i,t|r)=n_1).$$

The likelihood of a particular sequence of choices $\{X(i,t|r)\}_{t=1,\cdots,T_i}$ is simply

$$\begin{aligned}&l(\{X(i,t|r)\}_{t=1,\cdots,T_i})\\&=\prod_{t=1}^{T_i} q(X(i,t|r)|t-1,C_1(i,t-1|r),r).\end{aligned}\tag{C3}$$

Finally, assuming independence across sequences, the likelihood of observing a set of sequences $\{X(i,t|r)\}_{t=1,\cdots,T_i}, i\in I'$ is just

$$L(\{X(i,t|r)\}_{t=1,\cdots,T_i}, i\in I')=\prod_{i\in I'} l(\{X(i,t|r)\}_{t=1,\cdots,T_i}).\tag{C4}$$

For comparing models, we adopt AIC [37]. We denote the number of parameters of a model by N_p. AIC is defined using $\log L$ as

$$\mathrm{AIC}=-2\cdot\log L+2\cdot N_p.\tag{C5}$$